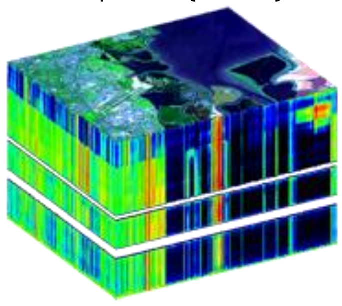
Hyperspectral Unmixing - Davor Josipovic

GHENT UNIVERSITY

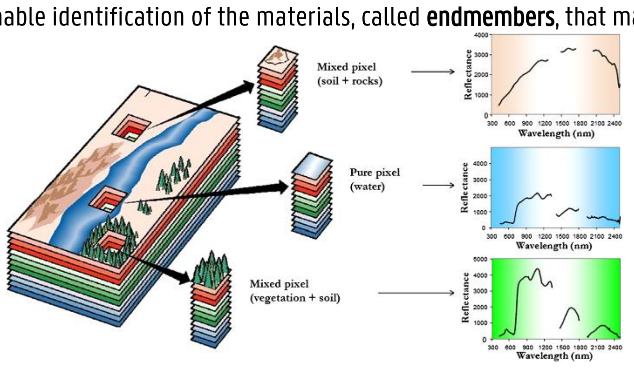
Hyperspectral Image

Where an ordinary image has only 3 bands (Red, Green, Blue), a hyperspectral has many more, ranging from hundreds to thousands. This hyperspectral data cube has slices showing the edges of the many bands ranging from the visible part of the spectrum (400 nm) to infrared (2.500 nm).



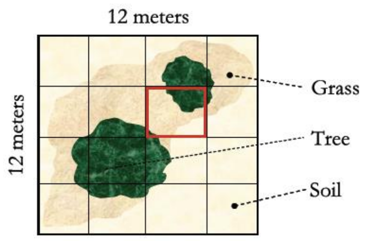
Endmember detection

Materials leave unique 'fingerprints' in the electromagnetic spectrum, known as spectral signatures. These signatures enable identification of the materials, called **endmembers**, that make up the image.



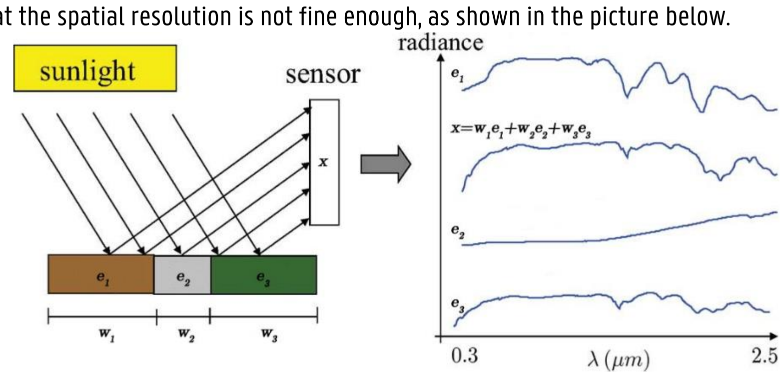
Spatial mixing

The emphasized (in red) pixel below is composed of 3 materials (i.e. endmembers) in the following **proportions**: 15% soil, 25% trees and 60% grass. We want to identify these endmembers given only the pixel spectra.



Linear mixing model

The linear mixing model (LMM) assumes that the incident light-rays interact only with the material (i.e. endmember) on which they scatter. The mixing thus occurs within the pixel-sensor itself due to the fact that the spatial resolution is not fine enough, as shown in the picture below.



LMM formalizes this composition of each pixel $\boldsymbol{x}_n$ by its constituent endmembers $\boldsymbol{e}_m$ according to the following formula:

$$\begin{aligned}\boldsymbol{x}_n &= w_{n1}\boldsymbol{e}_1 + \ \dots\ + w_{nM}\boldsymbol{e}_M + \boldsymbol{\epsilon} \\ &= \left(\sum_{m=1}^{M} w_{nm}\,\boldsymbol{e}_m\right) + \boldsymbol{\epsilon} \\ &= \boldsymbol{E}\boldsymbol{w}_n + \boldsymbol{\epsilon}\end{aligned}$$

under the constraints that $\|\boldsymbol{w}_n\|_1 = 1, w_{nm} \geq\ 0$ and $e_{bm} \geq 0\ \forall n, b, m$ where $\boldsymbol{e}_m$ are the endmember spectra and $\boldsymbol{w}_n$ the vector of proportions of each endmember in pixel $\boldsymbol{x}_n$.

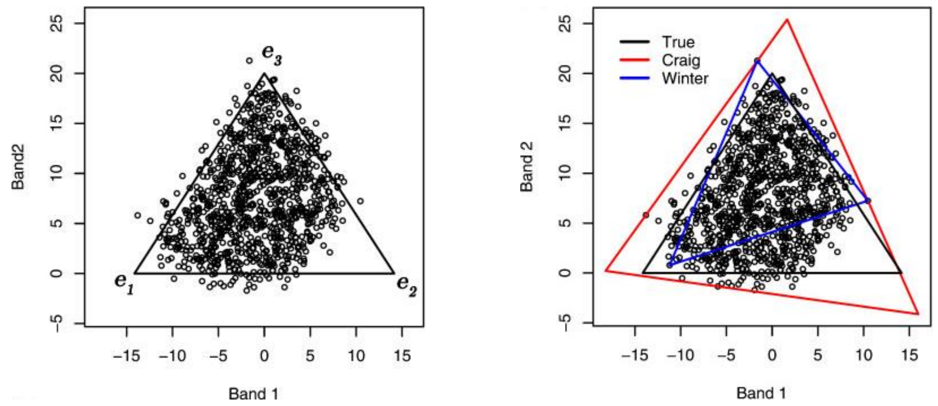
For all pixels we can write this in a more general form:

$$\boldsymbol{X} = \boldsymbol{W}\boldsymbol{E}^T + \boldsymbol{\epsilon}$$

Note that $\boldsymbol{W}, \boldsymbol{E}$ and $\boldsymbol{\epsilon} \sim \mathcal{N}(0, \sigma^2 \boldsymbol{I})$ are unknown. Note also that there are more unknowns than that there are pixel components x_{nb} and constraints $\|\boldsymbol{w}_n\|_1 = 1$ together.

Problem conceptualization

Suppose a synthetic image with 2 bands is generated based on 3 endmembers $\{e_1, e_2, e_3\}$ using LMM. The pixels are plotted below in function of band 1 and 2. The vertices of the black triangle are the true endmembers $\boldsymbol{e}_i$. *Devise an algorithm that can trace back the true endmembers $\boldsymbol{e}_i$.*



The red (Craig) and blue (Winter) simplexes are from the two of the best known algorithms for finding endmembers (i.e. the vertices of the simplex). But as one can see, in situations where some endmembers like $\boldsymbol{e}_2$ are not available in pure form, these algorithms do not perform well.

Solutions

Geometric approach

Minimize $L(\boldsymbol{W}, \boldsymbol{E}|\boldsymbol{X}) = \|\boldsymbol{X} - \boldsymbol{W}\boldsymbol{E}^T\|_F^2$. In this form, whenever all $\boldsymbol{X}$ lie within the simplex formed by vertices $\boldsymbol{E}$, the objective function $L(\cdot)$ will result in the same value, and thus problem is underspecified. Some regularization is needed. A popular regularization (Berman, 2003) is the volume V of the endmember simplex:

$$L(\boldsymbol{W}, \boldsymbol{E}|\boldsymbol{X}) = \|\boldsymbol{X} - \boldsymbol{W}\boldsymbol{E}^T\|_F^2 + \mu V$$

with μ a hyperparameter. Minimization is done with alternating least squares.

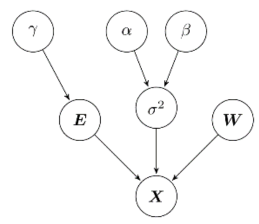
Model extension

We extended this model with an extra regularization S which favors higher correlation of abundances in neighboring pixels:

$$L(\boldsymbol{W}, \boldsymbol{E}|\boldsymbol{X}) = \|\boldsymbol{X} - \boldsymbol{W}\boldsymbol{E}^T\|_F^2 + \mu V + \nu S$$

Bayesian approach

The assumed DAG model with the posterior:



$$p(\boldsymbol{E}, \boldsymbol{W}, \sigma^2|\boldsymbol{X}, \alpha, \beta, \gamma) = \frac{p(\boldsymbol{X}|\boldsymbol{E}, \boldsymbol{W}, \sigma^2)p(\boldsymbol{E}|\gamma)p(\boldsymbol{W})p(\sigma^2|\alpha, \beta)}{p(\boldsymbol{X}|\alpha, \beta, \gamma)}$$

The prior $p(\boldsymbol{W})$ is assumed uninformative and $p(\sigma^2|\alpha, \beta) = inv.Gamma(\alpha, \beta)$, conjugate to the likelihood. The volume regularized prior $p(\boldsymbol{E}|\gamma) \propto\ e^{-\gamma V}$. Sampling is done with a self-implemented Gibbs sampler due to the sheer number of parameters and non-standard distributions.

Relation with the geometric approach

$$(\widehat{\boldsymbol{E}}, \widehat{\boldsymbol{W}})_{MAP} = \arg\min_{\boldsymbol{E},\boldsymbol{W}} \frac{\|\boldsymbol{X} - \boldsymbol{W}\boldsymbol{E}^T\|_F^2}{2\sigma^2} + \gamma V$$

Bayesian peculiarities

High variance? Inherent regularization even without $e^{-\gamma V}$?

You want to co-supervise?

References

- Arngren, Morten e.a., Bayesian nonnegative matrix factorization with volume prior for unmixing of hyperspectral images. In *2009 IEEE International Workshop on Machine Learning for Signal Processing*, pages 1-6. IEEE, 2009.
- Berman, Mark e.a., Ice: an automated statistical approach to identifying endmembers in hyperspectral images. In *Geoscience and Remote Sensing Symposium, 2003. IGARSS'03. Proceedings. 2003 IEEE International*, volume 1, pages 279-283. IEEE, 2003.